NORMANHURST BOYS HIGH SCHOOL
NEW SOUTH WALES

2013
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Student Number: _______________

Teacher: ________________________

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen
  Black pen is preferred
- Board- approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks (100)

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt questions 11 – 16
- Start a new booklet for each question
- Allow about 2 hours 45 minutes for this section

Students are advised that this is a school-based examination only and cannot in any way guarantee the content or format of future Higher School Certificate Examinations.

Section I

10 marks
Attempt all questions
Allow about 15 minutes for this section

Use the multiple choice answer sheet.

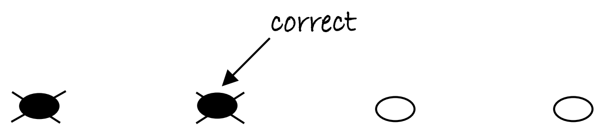
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A B C D

If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Section 1 (10 marks) **Marks**

(1) Which of the following is equal to 1.6 radians ? **1**

(A) $91^o40'$

(B) $[1.6 \times \pi]$ radians

(C) $\left[1.6 \times \frac{\pi}{180}\right]$ radians

(D) $12^o37'$

(2) In Mrs Lin's Year 6 class, 28 students play Badminton and 12 take chess as an activity. There are 33 students in the class and every student takes part in at least one of the activities mentioned. The probability that a particular student plays badminton and does not take chess as an activity is **1**

(A) $\frac{7}{10}$

(B) $\frac{28}{33}$

(C) 86%

(D) $\frac{7}{11}$

(3) The equation $x^2 + y^2 + 6y = 7$ describes a circle with: **1**

(A) $\text{Centre} = (0,3)$ and $\text{radius} = 4$

(B) $\text{Centre} = (0,-9)$ and $\text{radius} = 6$

(C) $\text{Centre} = (0,-3)$ and $\text{radius} = 4$

(D) $\text{Centre} = (0,3)$ and $\text{radius} = 6$.

Marks

(4) Which of the following values of m make the points $(4,-3)$, $(0,m)$ and $(-2,5)$ collinear? **1**

(A) $m=1$

(B) $m=\frac{7}{3}$

(C) $m=4$

(D) $m=-\frac{1}{2}$

(5) Given that $\log_a 3=x$ and $\log_a 2=y$, $\log_a 36$ can be written as **1**

(A) $2x+y$

(B) $2(x+y)$

(C) $(x+y)^2$

(D) $2xy$.

(6) The first three terms of a geometric series are $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\ldots$. If the series has a limiting sum, then **1**

(A) $r<-1 \text{ or } r>1$

(B) $-1<r<1$

(C) $|r|<1$

(D) None of the above.

(7) The roots of a quadratic are α and β. Given that $\alpha+\beta>0$, $\alpha^2+\beta^2=12$ *and* $\alpha\beta=2$, the equation of the quadratic could be written as **1**

(A) $x^2-8x+2=0$

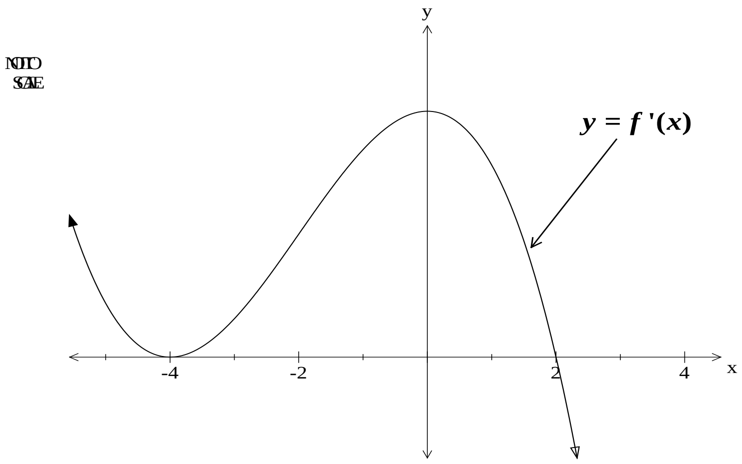
(B) $2x^2+8x+4=0$

(C) $x^2-4x+2=0$

(D) $x^2-7x+2=0$.

Marks

(8) **1**



The diagram above represents a sketch of the **gradient function** of the curve $y = f(x)$.
Which of the following is a true statement? The curve $y = f(x)$ has

(A) a minimum turning point occurs at $x = -4$

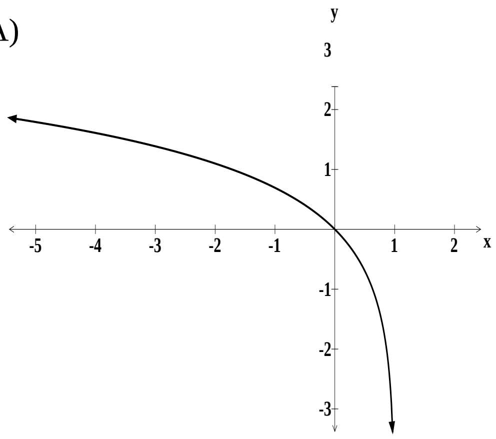
(B) a horizontal point of inflexion occurs at $x = 2$

(C) a horizontal point of inflexion occurs at $x = -4$

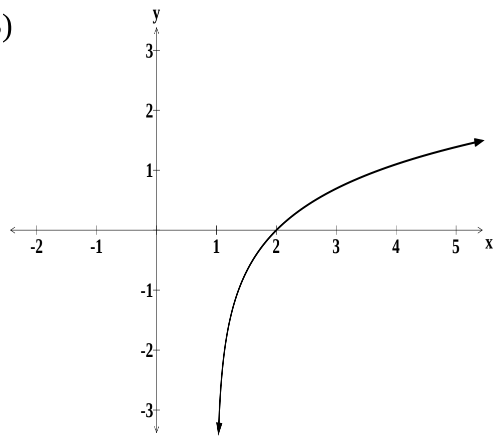
(D) a maximum turning point occurs at $x = 2$.

(9) Which of the following graphs represents $y = \ln(x+1)$? **1**

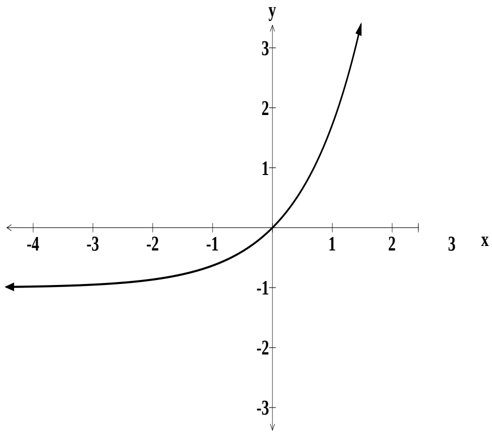
(A)



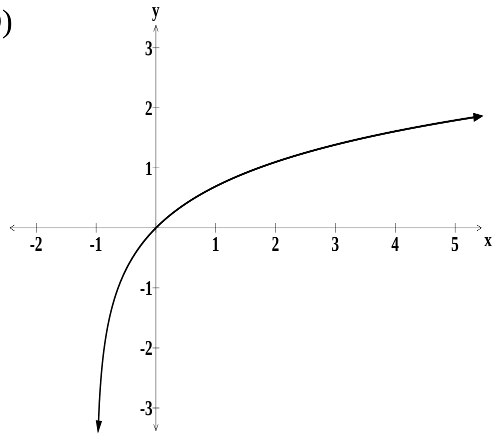
(B)



(C)



(D)



Marks

(10) Which of the following statements is mathematically correct? **1**

(A) An arithmetic progression can have a limiting sum.

(B) All cubic functions have a point of inflexion.

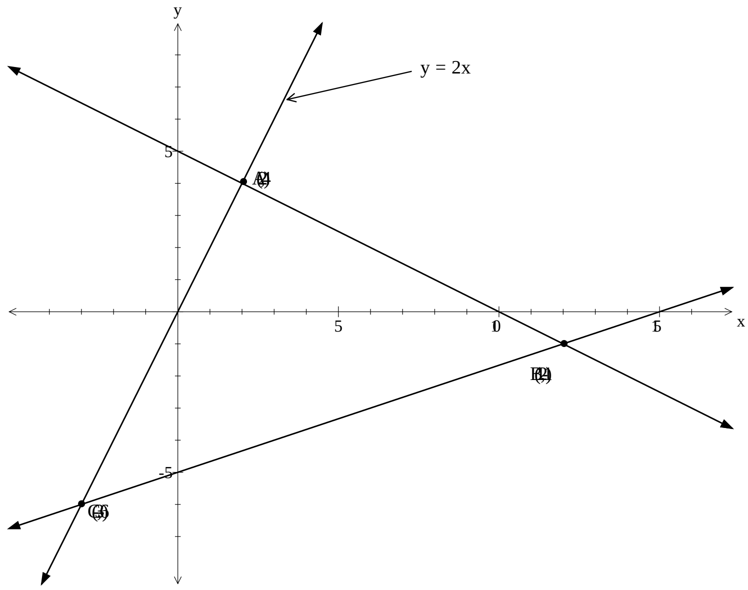
(C) Not all parabolas are symmetric.

(D) Every function has a corresponding inverse function.

Section II

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In the diagram above $A=(2,4), B=(12,-1)$ and $C=(-3,-6)$. A and C both lie on the line $y=2x$.

(i) Find the gradient of the line passing through *AB*. **1**

(ii) Prove that the equation of the line *AB* is $x+2y-10=0$. **2**

(iii) Prove that *AB* is perpendicular to *AC*. **1**

(iv) Prove that ΔABC is an isosceles triangle. **2**

(v) Hence or otherwise, find the area of ΔABC . **1**

(b) The interior angles of a regular polygon are 165^o each. Find the number of sides of this polygon. **2**

(c) (i) Show that $\dfrac{x+1}{x-1}=1+\dfrac{2}{x-1}$. **1**

(ii) Hence or otherwise graph $y=\dfrac{x+1}{x-1}$. **2**

(d) Find the coordinates of the point on the curve $y=2e^{3x}+1$, where the tangent to this curve is parallel to the line $12x-y+1=0$. **3**

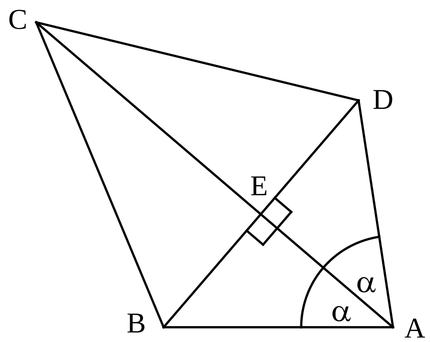
Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate the following with respect to *x*.

(i) $y = \cos^3 2x$. **2**

(ii) $y = \dfrac{e^{2x}}{2x+1}$. *(leave your answer in simplified form)* **3**

(b)



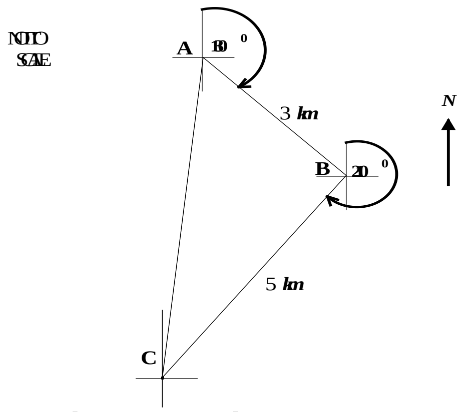
NOT TO SCALE

In the above diagram, *ABCD* is a quadrilateral where *AC* is perpendicular to *BD* and $\angle BAE = \angle DAE$.

(i) Prove that $\Delta ABE \equiv \Delta AED$. **2**

(ii) Hence or otherwise prove that ΔBCD is an isosceles triangle. **3**

(c)



A, *B* and *C* are markers in a yacht race. $AB = 3\,km$ and $BC = 5\,km$. The bearing of *B* from *A* is 130^oT and *C* from *B* is 210^oT.

Copy the diagram in your solution booklet

(i) Show clearly that $\angle ABC = 100^o$. **1**

(ii) Use the cosine rule to find the length of *AC*. *(to 2 decimal places)* **2**

(iii) Hence or otherwise, find the bearing of *A* from *C*. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) In a computer game Eduardo earned 40, 725 and 1050 points in each of the first three rounds respectively. If this pattern of numbers continues in all the next rounds find:

(i) The number of points that Eduardo earned in the 10^{th} round. **1**

(ii) The total number of points that Eduardo earned in all of the 1^{st} 10 rounds. **2**

(iii) How many rounds must Eduardo play to accumulate more than 50000 points in total ? **3**

(b) (i) Use the property that $\sin^2 x + \cos^2 x = 1$ to prove that **2**

$$\sec^2 x = 1 + \tan^2 x .$$

(ii) Hence or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \tan^2 x \; dx$. **2**

(c) Nam and Ambros each throw a normal six sided die.

(i) Find the probability that they throw the same number. **1**

(ii) Find the probability that the number thrown by Nam is smaller than the number thrown by Ambros. **2**

(iii) If the number thrown by each of them is multiplied together, find the probability that the result would be at least 20. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Given that $2\sin^2\theta - 5\sin\theta - 3 = 0$, find the exact value of θ, for $0 \le \theta \le 2\pi$. **3**

(b) At Nino's breakfast restaurant, the number (N) of customers in the restaurant at any time over a four hour period (t *hours*) is given by:

$$N = 4t^3 - t^4 + 20 \qquad 0 \le t \le 4$$

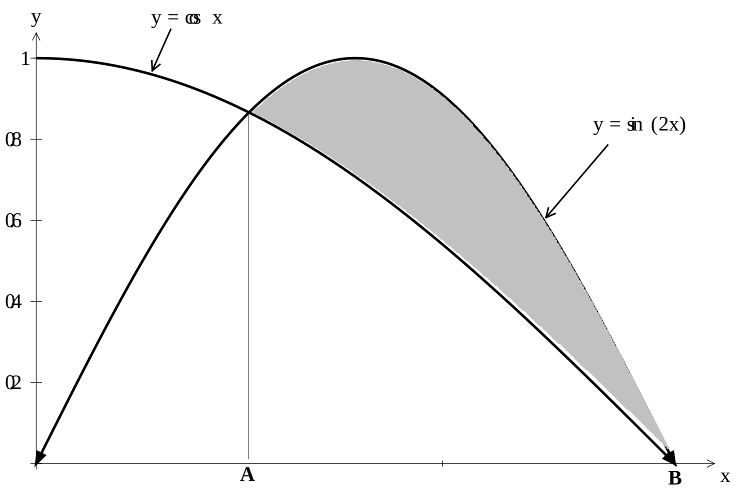
(i) Find the initial number of customers in the restaurant. **1**

(ii) Find the maximum number of customers in the restaurant during this time. **3**

(iii) Find the time when the number of customers was increasing most rapidly. **2**

(iv) Neatly sketch the curve $N = 4t^3 - t^4 + 20 \qquad 0 \le t \le 4$, showing all essential features. **2**

(c)



The graphs of $y = \cos x$ and $y = \sin 2x$ for $0 \le x \le \dfrac{\pi}{2}$ are represented above.

(i) Show that the x values of **A** and **B** (where the curves meet) are $\dfrac{\pi}{6}$ and $\dfrac{\pi}{2}$ respectively. **2**

(ii) Hence or otherwise, find the exact area of the shaded region. **2**

Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\lim_{x \to 2}\left(\dfrac{x^2-4}{x-2}\right)$ **1**

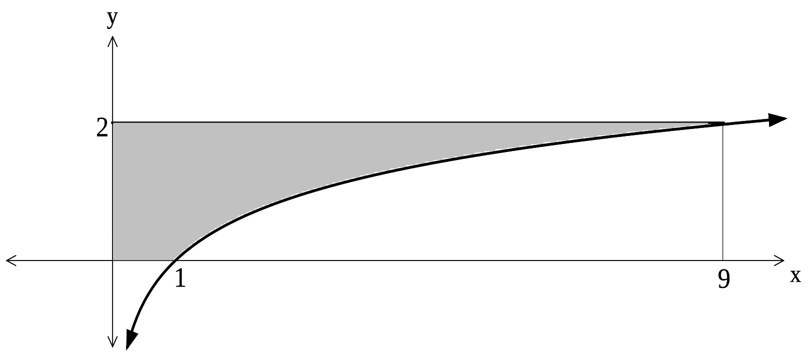
(b) Due to overfishing in a particular bay, the number (N) of a particular species of fish is dropping exponentially according to the formulae $\dfrac{dN}{dt} = -kN$, where time (t) is measured in years after 1930. It is known that in 1930 there were 25,000 fish of this species and by 2010 there were only 4000.

(i) Show that $N = Ae^{-kt}$, where A and k are constants. **1**

(ii) Find the value of A and show that $k = 0.0229$. **2**

(iii) This species of fish will be declared extinct in this bay when the number drops below 100 fish. In what year will this occur ? **2**

(c)



The above diagram shows the graph of $y = \log_3 x$ between $x = 0$ and $x = 9$. The shaded region, bounded by $y = \log_3 x$, the line $y = 2$ and the x and y axes, is rotated about the y-axis to form a solid.

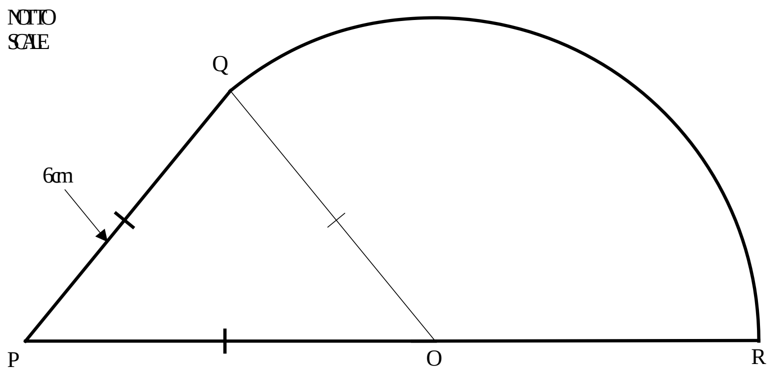
(i) Show that the volume of the solid is given by: **3**

$$V = \pi \int_0^2 \left(e^{y \ln 9}\right) dy$$

(ii) Hence find the volume of the solid, leaving your answer is simplified exact form. **2**

Question 15 continues over the page

(d) 15 (d)



ΔOPQ is an equilateral triangle of sides 6 cm. PR is a straight line. QR is an arc of a circle, centre O. Giving answers is exact form, find:

(i) The perimeter of the region $PQRO$. **2**

(ii) The area of the region $PQRO$. **2**

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Write down the domain of $y = \dfrac{1}{\sqrt{9 - x^2}}$ **2**

(b) (i) Use the Trapezoidal rule with three functions to find an approximation to the area under the curve $y = e^x$ between $x = a$ and $x = 5a$, where a is a positive number. **2**

(ii) Hence by rewriting the result in part (i), show that it can be written as $Area = ae^a \left[e^{2a} + 1\right]^2 \ units^2$ **1**

(c) Felix bought a second hand car for $30,000 with borrowed money from a finance company which charged him 18% p.a. reducible interest calculated monthly. Felix agreed to pay back the loan plus interest at $900 per month.

(i) Show that the amount that Felix owed after his second payment was $A_2 = 30,000(1.015)^2 - 900(1 + 1.015)$. **1**

(ii) Show that the amount owing after n payments have been made can be expressed as $A_n = 60,000 - 30,000(1.015)^n$ **3**

(iii) Hence find the number of months that Felix required to pay back the loan plus interest. **1**

(d) A particle is moving in a straight line. Its displacement from the origin (*x cm*) as a function of time (*t minutes*) is given by $x = t\sin t + \cos t$.

(i) Show that $v = t\cos t$. **1**

(ii) Hence find the 1st four occasions when the particle changes direction. **2**

(iii) Show that $t = \dfrac{\pi}{2} + 2n\pi$ (*where n is an integer*) are the occasions when the particle changes from a positive to a negative direction. **2**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2013 Trial HSC Mathematics Solutions

Section II

Question 11 (15 marks)

11(a) (i) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

$$m=\frac{4+1}{2-12}$$
$$=-\frac{1}{2}$$

11(a) (ii) (2 marks)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Substitutes into correct formulae	1
• Correct working	1

Answer

$$y-4=-\frac{1}{2}(x-2)$$
$$2y-8=-x+2$$
$$x+2y-10=0$$

11(a) (iii) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct working	1

Answer

$$m_{AB}=-\frac{1}{2}$$
$$m_{AC}=\frac{4+6}{2+3}$$
$$=2$$
$$m_{AB}.m_{AC}$$
$$=-\frac{1}{2}\times 2$$
$$=-1$$
$\therefore AB \perp AC$

11(a) (iv) (2 marks)

Outcomes Assessed: P4, H2

Targeted Performance Bands: 3-4

Criteria	Marks
• Uses the distance formulae once correctly	1
• Correct proof	1

Answer

$$\overline{AB}=\sqrt{100+25}$$
$$=\sqrt{125}=5\sqrt{5}$$
$$\overline{AC}=\sqrt{25+100}$$
$$=\sqrt{125}=5\sqrt{5}$$

Since $\overline{AB}=\overline{AC}$ $\therefore \Delta ABC$ is isosceles

11(a) (v) (1 mark)

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

$$\text{Area}=\left[\frac{1}{2}\times\overline{AC}\times\overline{AB}\right]$$
$$=\frac{1}{2}\left[5\sqrt{5}\times5\sqrt{5}\right]$$
$$=62\frac{1}{2}\ units^2$$

11(b) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct working	1
• Correct answer	1

Answer

$$\text{Each exterior angle}=180^\circ-165^\circ$$
$$=15^\circ$$
$$\text{number of sides}=\frac{360}{15}$$
$$=24\text{ sides}$$

11(c) (i) (1 mark)

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct working	1

Answer

$\frac{x+1}{x-1}=1+\frac{2}{x-1}$

$LHS=\frac{x-1+2}{x-1}$

$\frac{x-1}{x-1}+\frac{2}{x-1}$

$=1+\frac{2}{x-1}$

$=RHS$

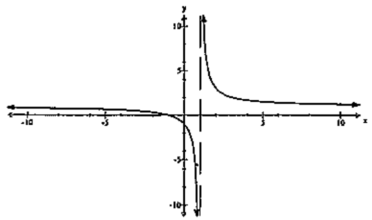
11(c) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct asymptotes	1
• Correct graph	1

Answer



11(d) (3 marks)

Outcomes Assessed: H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Differentiates correctly	1
• Achieves $x=\frac{\ln 2}{3}$	1
• Correct answer	1

Answer

$y'=6e^{3x} \quad m=12$

$\therefore 6e^{3x}=12$

$e^{3x}=2$

$3x=\ln 2$

$x=\frac{\ln 2}{3}$

$f\left(\frac{\ln 2}{3}\right)=2e^{3\left(\frac{\ln 2}{3}\right)}+1$

$=5$

$\text{Pt}=\left(\frac{\ln 2}{3},5\right)$

Question 12 (15 marks)

12(a) (i) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Either uses the chain rule correctly or differentiates $\cos 2x$ correctly	1
• Correct answer	1

Answer

$y=\cos^3 2x$

$y'=-6\cos^2 2x\sin 2x$

12(a) (ii) (3 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 2-4

Criteria	Marks
• Uses the quotient rule (or product rule after rewriting)	1
• Correct working with one mistake	1
• Correct answer	1

Answer

$$y=\frac{e^{2x}}{2x+1}$$

$$y'=\frac{(2x+1).2e^{2x}-2e^{2x}}{(2x+1)^2}$$

$$=\frac{4xe^{2x}}{(2x+1)^2}$$

12(b) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Uses correct test	1
• Correct proof	1

Answer

In ΔABE & ΔAED

$\angle BAE=\angle DAE$ (given)

$\angle DEA=\angle BEA$ (90°, given)

AE is common

$\Delta ABE\equiv\Delta AED$ (A.A.S)

12(b) (ii) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Show BE=ED	1
• Prove $\Delta BCE\equiv\Delta CDE$	1
• Correct proof	1

Answer

In ΔBCE & ΔCDE

$BE=ED$ (corresponding sides of congruent triangles ABE & AED)

$\angle BEC=\angle DEC$ (90°, given)

EC is common

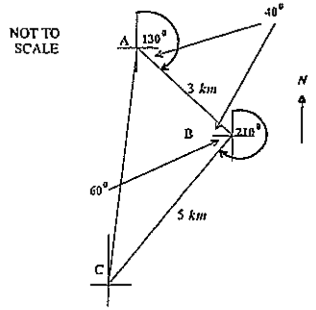
$\Delta BCE\equiv\Delta CDE$ (S.A.S)

$\therefore \Delta BCD$ is isosceles ($CB=CD$, corresponding sides of congruent triangles)

12(c) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly working	1

Answer



$\angle ABC=40°+60°=100°$

12(c) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Uses the cosine rule correctly	1
• Correct answer	1

Answer

$AC^2 = 3^2 + 5^2 - 2\times3\times5\times\cos100^o$

$AC = 6.26\,km$

12(c) (iii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-4

Criteria	Marks
• Obtains $\angle ACB$	1
• Correct answer	1

Answer

Let $\angle ACB = \theta$

$\frac{\sin\theta}{3} = \frac{\sin100^o}{6.26}$

$\sin\theta = \frac{3\sin100^o}{6.26}$

$\theta = 28^o$

$\therefore$ *Bearing of A from C* $= 90^o - [28^o + 60^o]$

$= 2^oT$

Question 13 (15 marks)
13(a) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Mark
• Correct working and answer	2

Answer

$T_n = a + (n-1)d$

$T_{10} = 400 + (9)325$

$= 3325$ points

13(a) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 2-3

Criteria	Marks
• Correct working	1
• Correct answer	1

Answer

$S_n = \frac{n}{2}\{a + l\}$

$S_{10} = \frac{10}{2}\{400 + 3325\}$

$= 18625$ points

13(a) (iii) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct working	1
• Obtains correct quadratic	1
• Correct answer	1

Answer

$\frac{n}{2}\{2a + (n-1)d\} = 50000$

$\frac{n}{2}\{800 + (n-1)325\} = 50000$

$\frac{n}{2}\{325n + 475\} = 50000$

$325n^2 + 475n - 100000 = 0$

$13n^2 + 19n - 4000 = 0$

$n = \frac{-19 \pm \sqrt{208361}}{26}$

$\doteqdot 16.8256$, as $n > 0$

Eduardo must play 17 games.

13(b) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Bands: 4-5

Criteria	Marks
• ~~Works towards answer~~	
• Correct proof	1

Answer

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

13(b) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct integration	1
• Correct answer	1

Answer

$$\int_0^{\frac{\pi}{4}} \tan^2 x \quad dx = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 \quad dx$$

$$= \left[\tan x - x\right]_0^{\frac{\pi}{4}}$$

$$= \left(1 - \frac{\pi}{4}\right)$$

13 (c) (i) (1 mark)

Outcomes Assessed: H5

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

$$P(E) = \frac{6}{36}$$

$$= \frac{1}{6}$$

13(c) (ii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Notes that there will be 15 occasions when this will occur	1
• Correct answer	1

Answer

$$P(E) = \frac{15}{36}$$

$$= \frac{5}{12}$$

13(c) (iii) (2 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Notes that there will be 8 occasions when this will occur	1
• Correct answer	1

Answer

$$P(E) = \frac{8}{36}$$

$$= \frac{2}{9}$$

Question 14 (15 marks)

14(a) (3 marks)

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly factors quadratic	1
• Obtains one answer $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$	1
• Correct answer	1

Answer

$$2\sin^2\theta - 5\sin\theta - 3 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 3) = 0$$

$$\sin\theta = -\frac{1}{2} \qquad \sin\theta = 3$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad \theta = \text{no solutions}$$

$$\therefore \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

14(b) (i) (1 mark)

Outcomes Assessed: H1

Targeted Performance Bands: 2-3

Criteria	Mark
• Correct answer	1

Answer

$t=0 \quad N=20$

14(b) (ii) (3 marks)

Outcomes Assessed: H6

Targeted Performance Bands: 3-4

Criteria	Marks
• Obtain stationary points	1
• Determines their nature	1
• Correct answer	1

Answer

$N=4t^3-t^4+20$

$N'=12t^2-4t^3$

$N''=24t-12t^2$

Stationary points occur when $N'=0$

$12t^2-4t^3=0$

$3t^2-t^3=0$

$t^2(3-t)=0$

$t=0,3$

$N(0)=20$

$N(3)=108-81+20=47$

Check concavity:

$N''(0)=0 \qquad \therefore (0,0)$ is a possible P.I.

$N''(3)=-36 \qquad \therefore (3,47)$ is a maximum

Max number of customers = 47.

14(b) (iii) (2 marks)

Outcomes Assessed: H6

Targeted Performance Bands: 5-6

Criteria	Marks
• Solves $N''(t)=0$	1
• Correct answer & shows $N'(t)$ changes sign around $t=2$	1

Answer

Number of customers increase most rapidly when $N''=0$

$24t-12t^2=0$

$2t-t^2=0$

$t(2-t)=0$

$t=0,2$

Check:

t	-1	0	1	2	3
N'	-	0	12	0	-36

Therefore (0, 0) and (2,36) are points of inflexion.

At t=2 is the time when the number of customers increase most rapidly $N'(2)=+$

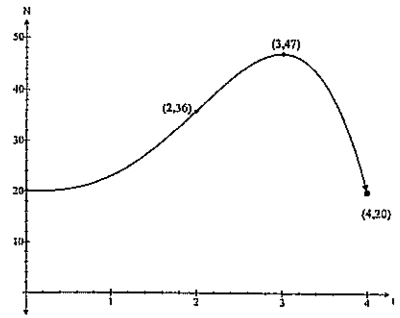
14(b) (iv) (2 marks)

Outcomes Assessed: H6

Targeted Performance Bands: 4-5

Criteria	Marks
• Notes turning point and point of inflexion	1
• Correct graph	1

Answer



14(c) (i) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 2-3

Criteria	Marks
• Correct working for A	1
• Correct working for B (note for Ext 1 students achieve answers by solving $\cos x = \sin 2x$)	1

Answer

$y = \cos x$

$$f\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$y = \sin 2x$

$$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$f\left(\frac{\pi}{2}\right) = \sin(\pi) = 0$$

14(c) (ii) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct integration	1
• Correct answer	1

Answer

$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x - \cos x \ dx$$

$$= \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -\left[\frac{1}{2}\cos 2x + \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -\left[\left(-\frac{1}{2}+1\right) - \left(\frac{1}{4}+\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{4}\right]$$

$$= \frac{1}{4} \text{ sq. units}$$

Question 15 (15 marks)

15(a) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct answer	1

Answer

$$\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right) = \lim_{x\to 2}\left(\frac{(x-2)(x+2)}{(x-2)}\right)$$

$$= \lim_{x\to 2}(x+2)$$

$$= 4$$

15(b) (i) (1 mark)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct proof	1

Answer

$$N = Ae^{-kt}$$

$$N' = -k\left(Ae^{-kt}\right)$$

$$\frac{dN}{dt} = -kN \quad \left(\text{since } N = Ae^{-kt}\right)$$

OR $\int \frac{dN}{N} = \int -k\,dt$

⋮

15(b) (ii) (2 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct A	1
• Correct working for k	1

Answer

$$A = 25{,}000$$

$$4000 = 25000e^{-k(80)}$$

$$0.16 = e^{-80k}$$

$$\ln(0.16) = -80k$$

$$k = \frac{\ln(0.16)}{-80}$$

$$k = 0.0229$$

15(b) (iii) (2 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 3-4

Criteria	Marks
• Achieves $t = 241.11\,years$	1
• Correct answer ≈ 2172	1

Answer

$$100 = 25000e^{-kt}$$
$$0.004 = e^{-0.0229t}$$
$$\ln(0.004) = -0.0229t$$
$$t = \frac{\ln(0.004)}{-0.0229}$$
$$t = 241.11\,years$$

Therefore require 242 years occurring in 2172.

15(c) (i) (3 marks)
Outcomes Assessed: H3
Targeted Performance Bands: 5-6

Criteria	Marks
• ~~Change base~~ Achieves x	1
• Achieves $\left(e^{y\ln 3}\right)^2$	1
• ~~Correct working~~ $V = \pi\int_0^2 x^2\,dy$	1

Answer

$$y = \log_3 x$$
$$y = \frac{\ln x}{\ln 3}$$
$$y\ln 3 = \ln x$$
$$x = e^{y\ln 3} \checkmark$$
$$\therefore x^2 = \left(e^{y\ln 3}\right)^2$$
$$= e^{2y\ln 3}$$
$$= e^{y\ln 3^2}$$
$$= e^{y\ln 9} \checkmark$$
$$V = \pi\int_0^2 x^2\,dy \checkmark = \pi\int_0^2 e^{y\ln 9}\,dy$$

OR

$$x = 3^y \checkmark$$
$$x^2 = (3^y)^2$$
$$= 3^{2y}$$
$$= 9^y$$
$$= e^{\ln 9^y}$$
$$= e^{y\ln 9} \checkmark$$
$$V = \pi\int x^2\,dy \checkmark$$

Note $\left(e^{y\ln 3}\right)^2 = e^{y\ln 9}$ −1 mark

15(c) (ii) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 4-5

Criteria	Marks
• Correct integration	1
• Correct answer	1

Answer

$$V = \pi\int_0^2 \left(e^{y\ln 9}\right) dy$$
$$= \frac{\pi}{\ln 9}\left[e^{y\ln 9}\right]_0^2$$
$$= \frac{\pi}{\ln 9}[81-1]$$
$$= \frac{80\pi}{\ln 9}\,units^3$$

* CFPA requires substitution into $e^{y\ln 9}$ and simplify

15(d) (i) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• ~~Correct working~~	1
• Correct answer	1

Answer

$$\angle POQ = 60^\circ$$
$$\therefore \angle QOP = 120^\circ = \frac{2\pi}{3}$$
$$\therefore Perimeter = 3(6) + 6\left(\frac{2\pi}{3}\right)$$
$$= 18 + \frac{12\pi}{3}$$
$$= (18 + 4\pi)\,units$$

15(d) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct working	1
• Correct answer	1

Answer

$$Area = \left(\frac{1}{2}\times 6\times 6\times \sin\frac{\pi}{3}\right)+\frac{1}{2}(6)^2\left(\frac{2\pi}{3}\right)$$
$$=\left(9\sqrt{3}+12\pi\right)units^2$$

Question 16 (15 marks)
16(a) (2 marks)
Outcomes Assessed: H9
Targeted Performance Bands: 4-5

Criteria	Marks
• Notes that $9-x^2>0$	1
• Correct answer	1

Answer

$9-x^2>0$
$x^2-9<0$
$-3<x<3$
$\therefore Domain=\{x:-3<x<3\}$

16(b) (i) (2 marks)
Outcomes Assessed: H8
Targeted Performance Bands: 4-5

Criteria	Marks
• Uses the trapezoidal rule correctly with one mistake	1
• Correct answer	1

Answer

x	y	weight	Result
a	e^a	1	e^a
3a	e^{3a}	2	$2e^{3a}$
5a	e^{5a}	1	e^{5a}
Total			$e^a+2e^{3a}+e^{5a}$

$$A=\frac{2a}{2}\left[e^a+2e^{3a}+e^{5a}\right]$$
$$=a\left[e^a+2e^{3a}+e^{5a}\right]units^2$$

-1 mark for NOT showing clear application of Trapezoidal rule

16(b) (ii) (1 mark)
Outcomes Assessed: H3
Targeted Performance Bands: 4-5

Criteria	Mark
• Correct working	1

Answer

$$A=a\left[e^a+2e^{3a}+e^{5a}\right]$$
$$=a\left[e^a\left(1+2e^{2a}+e^{4a}\right)\right]$$
$$=ae^a\left(e^{2a}+1\right)^2$$

16 (c) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct working	1

Answer

$A_1=30{,}000(1.015)^1-900$
$A_2=\left[30{,}000(1.015)^1-900\right](1.015)^1-900$
$=30{,}000(1.015)^2-900(1+1.015)$

-1 mark if
$A_2=30000(1.015)^2-900(1.015)-900$
$=30000(1.015)^2-900(1+1.015)$

16(c) (ii) (3 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4-6

Criteria	Marks
• Achieves $A_n = 30,000(1.015)^n - 900(1+1.015+1.015^2+.....1.015^{n-1})$	1
• Calculates sum of a G.P.	1
• ~~Correct working~~	~~1~~

Answer

$$A_3 = \{30,000(1.015)^2 - 900(1+1.015)\}(1.015)^1 - 900$$
$$= 30,000(1.015)^3 - 900(1+1.015+1.015^2)$$
$$\therefore A_n = 30,000(1.015)^n - 900(1+1.015+1.015^2+.....1.015^{n-1})$$
$$= 30,000(1.015)^n - 900\left[\frac{1(1.015^n - 1)}{.015}\right]$$
$$= 30,000(1.015)^n - 60,000[(1.015^n - 1)]$$
$$= 60,000 - 30,000(1.015)^n$$

16(c) (iii) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 4-5

Criteria	Mark
• Correct $n = 46.55$	1
Correct statement i.e. 47 months	1

Answer

$$60,000 - 30,000(1.015)^n = 0$$
$$60,000 = 30,000(1.015)^n$$
$$2 = (1.015)^n$$
$$\ln 2 = n\ln(1.015)$$
$$n = \frac{\ln 2}{\ln(1.015)}$$
$$n = 46.55 \textit{ months}$$

Felix will require 47 months *(the 47th payment will be part of $900).*

16(d) (i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correct working	1

Answer

$$x = t\sin t + \cos t$$
$$v = \sin t \times 1 + t \times \cos t - \sin t$$
$$= t\cos t$$

16(d) (ii) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Notes that $v = 0$ and achieves at least one answer	1
• Correct answer ie all four correct answers	2

Answer

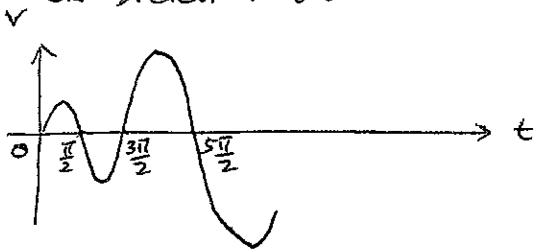
$$v = 0$$
$$t\cos t = 0$$
$$t = 0 \qquad \cos t = 0$$
$$t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$
$$\therefore t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

16(d) (iii) (2 marks)
Outcomes Assessed: H1, H5, H9
Targeted Performance Bands: 5-6

Criteria	Marks
• Calculates which values of t provide a maximum	1
• Correct working	1

Answer

OR sketch $v = t\cos t$



$\ddot{x} = \cos t - t\sin t$

$\ddot{x}(0) = 1$ minimum turning point

$\ddot{x}\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$ maximum turning point

$\ddot{x}\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2}$ minimum turning point

$\ddot{x}\left(\frac{5\pi}{2}\right) = -\frac{5\pi}{2}$ maximum turning point

Hence, maximum turning points occur when $t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots\ldots$ *i.e. when* $t = \frac{\pi}{2} + 2n\pi$ *for n an integer.*

Section I

Question	Marks	Answer
1	1	A
2	1	D
3	1	C
4	1	B
5	1	B
6	1	A
7	1	C
8	1	C
9	1	D
10	1	B

Trial HSC 2013 Mathematics

Student Number:__________

Multiple Choice Answer Sheet

Teacher:__________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ● B ⊗ ← correct C ○ D ○

1. A ● B ○ C ○ D ○
2. A ○ B ○ C ○ D ●
3. A ○ B ○ C ● D ○
4. A ○ B ● C ○ D ○
5. A ○ B ● C ○ D ○
6. A ● B ○ C ○ D ○
7. A ○ B ○ C ● D ○
8. A ○ B ○ C ● D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ● C ○ D ○